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Seismic response analysis of structures using time-step group integration method

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ABSTRACT

This paper presents a higher-order accurate time-step group integration method for analyzing the seismic response of structures. Unlike traditional step-by-step procedures, the proposed method treats a group of p (p ≥ 2) consecutive time steps as a unified computational interval. By establishing a time-step group-by-group integration strategy, the differential equations to earthquake-induced ground motion are solved to obtain all p time-step solutions simultaneously through matrix operations. Crucially, this method avoids the complex integral computations in theoretical solutions by first deriving the time derivatives of state response vectors, then converting these derivatives into seismic responses through differential quadrature transformation. These unknown steps within each time-step group serve not only as the variables to be solved but also as the foundation for constructing a higher-order accurate integration scheme, so there is no need to set additional sub-steps nor to assume the specific variation of accelerations, velocities, and displacements within the predefined time intervals. Numerical analysis of a three-story shear frame demonstrates that the proposed method can achieve improved accuracy with stability and controllable numerical dissipation, especially for long-term seismic response analysis problems, it can still obtain the accurate computational results.

1. INTRODUCTION

Accurate simulation of structural seismic responses demands time integration algorithms that concurrently achieve high precision, numerical stability, and computational efficiency, which magnified undoubtedly by the non-stationary nature of earthquake excitations. While second-order methods like Newmark-β (Newmark 1959) and HHT-α (Hilber, et al. 1977) remain industry standards due to their unconditional stability, their inherent limitations in accuracy (Bathe, et al. 1972) and excessive numerical damping (Hulbert, 1992) critically distort long-duration seismic analyses. Such distortions escalate risks in assessing cumulative damage mechanisms, particularly for structures subjected to near-fault pulse-type ground motions where high-frequency

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components dominate (Kalkan, et al. 2007).

High-order accurate time integration methods (order ≥4) emerge as essential tools to mitigate these errors. Theoretical advances, including composite sub-step schemes (Dormand, et al. 1980) and symplectic algorithms (Simo, et al. 1992), demonstrate that fourth-order precision reduces period errors and amplitude decay. However, their adoption in seismic engineering remains limited by two fundamental barriers: one is the computational burden of iterative sub-step calculations, which inflates runtime compared to Newmark-β (Soares 2016), and the other is the intractable evaluation of Duhamel integral terms under non-analytic ground accelerations — a universal challenge as closed-form solutions vanish for real-world seismic inputs.

Recent efforts to bypass these barriers fall into two categories. Frequency-domain methods (Clough, et al. 2003) transform dynamic equations into algebraic forms via Fourier transforms, yet their accuracy plummets for nonlinear systems common in post-yielding seismic responses. Group-based strategies like parallelizable composite schemes (Fung 2001) process multiple time steps as computational blocks, but still rely on numerical quadrature (e.g., Gauss-Legendre rules) for convolution integrals. This persistent dependence on integral evaluations — whether direct or indirect — anchors a critical research gap, namely no existing high-order method fully decouples solution accuracy from numerical integration errors in seismic applications.

This study introduces a time-step group integration method (TGIM) that concurrently addresses these limitations. By redefining p consecutive time steps as a unified solvable entity, the method achieves p-order accuracy without sub-step divisions. Its core innovation lies in replacing traditional integral computations with derivative-to-response transformations, effectively decoupling solution accuracy from numerical quadrature errors. Validated against Newmark-β benchmarks, the proposed method reduces computational effort significantly while maintaining stability at time steps larger than conventional approaches.

2. SEISMIC RESPONSE OF STRUCTURES

The dynamic response of a structure under seismic ground motion can be decomposed into the superposition of modal responses. Thus, seismic response analysis can be accomplished through modal analysis. For a classically damped multi-degree-of-freedom system, the response under the *j*-th vibration mode is expressed as

$$m_j\ddot{q}_j(t)+c_j\dot{q}_j(t)+k_jq_j(t)=-m_j\gamma_j\ddot{u}_g(t), \tag{1}$$

where m_j, c_j, and k_j represent the modal mass, modal damping, and modal stiffness of the *j*-th mode, respectively; $q_j(t)$ denotes the generalized coordinate of the *j*-th mode, with $\dot{q}_j(t)$ and $\ddot{q}_j(t)$ being its first and second time derivatives; γ_j is the modal participation factor, and $\ddot{u}_g(t)$ represents the ground acceleration time history.

The time history is discretized into sequential time steps of equal duration Δt. Assuming the solutions of Eq. (1) up to time t_i have been obtained, we consider a time-step group containing *p* subsequent steps starting from t_i, as illustrated in Fig. 1. The interval of this group is [t_i, t_{i+p}], with its length *h* related to the time step size as

$$h = p\Delta t, \tag{2}$$

within [t_i, t_{i+p}], the solution to Eq. (1) can be expressed as

$$q(t) = q_c(t) + q_p(t), \tag{3}$$

where

$$q_c(t) = \mathrm{e}^{-\zeta\omega(t-t_i)}\left[q(t_i)\cos\omega_D(t-t_i) + \frac{\dot{q}(t_i)+\zeta\omega q(t_i)}{\omega_D}\sin\omega_D(t-t_i)\right], \qquad t_i \le t \le t_{i+p}, \tag{4}$$

$$q_p(t) = -\frac{\gamma}{\omega_D}\int_0^{t-t_i}\ddot{u}_g(t_i+\theta)\mathrm{e}^{-\zeta\omega(t-t_i-\theta)}\sin\omega_D(t-t_i-\theta)\mathrm{d}\theta, \qquad t_i \le t \le t_{i+p}, \tag{5}$$

Here, ω and ζ denote the modal frequency and damping ratio ($\omega = \sqrt{k/m}$), respectively, and $\omega_D = \omega\sqrt{1-\zeta^2}$ represents the damped modal frequency. The subscript *j* indicating mode number is omitted for simplicity.

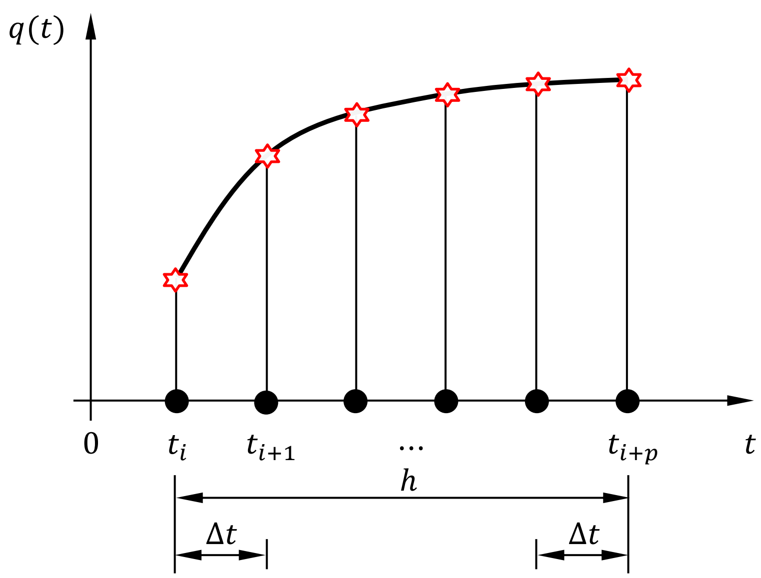

Fig. 1 The time-step group for seismic response of structures

Since both the displacement $q(t_i)$ and velocity $\dot{q}(t_i)$ have been obtained from previous computations, $q_c(t)$ in Eq. (4) can be directly determined. For the integral term $q_p(t)$ in Eq. (5), by separating time-independent components from the integrand, the expression can be reformulated as

$$q_p(t) = -\frac{\gamma \mathrm{e}^{-\zeta\omega(t-t_i)}}{\omega_D}\left[\sin\omega_D(t-t_i)R_c(t-t_i) - \cos\omega_D(t-t_i)R_s(t-t_i)\right], \qquad t_i \le t \le t_{i+p}, \tag{6}$$

where the functions R_c and R_s are variable upper limit integrals defined as

$$\begin{cases} R_c(\tau) = \int_0^\tau \ddot{u}_g(t_i+\theta)\mathrm{e}^{\zeta\omega\theta}\cos\omega_D\theta\mathrm{d}\theta \\ R_s(\tau) = \int_0^\tau \ddot{u}_g(t_i+\theta)\mathrm{e}^{\zeta\omega\theta}\sin\omega_D\theta\mathrm{d}\theta \end{cases}, \tag{7}$$

The seismic response time history is then obtained by substituting the solutions of these integrals into Eq. (6) and subsequently applying Eq. (3).

3. TIME-STEP GROUP INTEGRATION METHOD

The computation of the two integrals in Eq. (7) is pivotal for solving Eq. (1), yet direct numerical evaluation of these integrals proves challenging. To address this, we first differentiate $R_c(\tau)$ and $R_s(\tau)$:

$$\dot{R}_c(\tau) = \frac{\mathrm{d}}{\mathrm{d}\tau}R_c(\tau) = \ddot{u}_g(t_i+\tau)\mathrm{e}^{\zeta\omega\tau}\cos\omega_D\tau, \tag{8a}$$

$$\dot{R}_s(\tau) = \frac{\mathrm{d}}{\mathrm{d}\tau}R_s(\tau) = \ddot{u}_g(t_i+\tau)\mathrm{e}^{\zeta\omega\tau}\sin\omega_D\tau, \tag{8b}$$

where τ serves as a local coordinate within the interval [t_i, t_{i+p}]. For uniform time steps Δt, the relationship satisfies

$$\tau_k = t_{i+k} - t_i = k\cdot\Delta t \quad \text{for} \quad k = 0, 1, \cdots, p, \tag{9}$$

Applying the differential quadrature (DQ) principle, the derivative of R_c over [t_i, t_{i+p}] (or [0, τ_p] in the local coordinate) is approximated as a weighted linear combination of its function values:

$$\dot{R}_n(\tau_k) = \sum_{j=0}^{p} a_{kj}R_n(\tau_j) \quad \text{for} \quad k = 1, 2, \cdots p, \quad n = c, s, \tag{10}$$

where a_{kj} denotes DQ weighting coefficients determined solely by the coordinate of τ_k. For uniform steps, these coefficients are calculated as

$$a_{kj} = (-1)^{j-k} \cdot \frac{k!(p-k)!}{(k-j)\cdot j!(p-j)!} \cdot \frac{1}{\Delta t}, \tag{11}$$

Substituting Eq. (9) into Eq. (10) yields the matrix formulation:

$$\begin{Bmatrix} \dot{R}_n(\Delta t) \\ \dot{R}_n(2\Delta t) \\ \vdots \\ \dot{R}_n(p\Delta t) \end{Bmatrix} = \begin{Bmatrix} a_{10} \\ a_{20} \\ \vdots \\ a_{p0} \end{Bmatrix} R_n(0) + \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ a_{p1} & a_{p2} & \cdots & a_{pp} \end{bmatrix} \begin{Bmatrix} R_n(\Delta t) \\ R_n(2\Delta t) \\ \vdots \\ R_n(p\Delta t) \end{Bmatrix} \quad \text{or} \quad \dot{\boldsymbol{R}}_n = \boldsymbol{A}_0 R_n(0) + \boldsymbol{A}\boldsymbol{R}_n, \tag{12}$$

Noting $R_c(0) = 0$ from Eq. (7), Eq. (12) is solved as

$$\boldsymbol{R}_n = \boldsymbol{T} \cdot \dot{\boldsymbol{R}}_n, \tag{13}$$

where the transformation matrix $\boldsymbol{T}$ is derived from

$$\boldsymbol{T} = \boldsymbol{A}^{-1} = \boldsymbol{G} \cdot \boldsymbol{V}^{-1}, \tag{14}$$

Here, $\boldsymbol{V}$ is a Vandermonde matrix and $\boldsymbol{G}$ a square matrix defined by

$$\boldsymbol{G} = \begin{bmatrix} 1 & \Delta t/2 & \cdots & \Delta t^{p-1}/p \\ 2 & 2\Delta t & \cdots & 2^p \cdot \Delta t^{p-1}/p \\ \vdots & \vdots & \ddots & \vdots \\ p & p^2 \cdot \Delta t/2 & \cdots & p^{p-1} \cdot \Delta t^{p-1} \end{bmatrix}, \quad \boldsymbol{V} = \begin{bmatrix} 1 & \Delta t & \cdots & \Delta t^{p-1} \\ 1 & 2\Delta t & \cdots & (2\Delta t)^{p-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & p\Delta t & \cdots & (p\Delta t)^{p-1} \end{bmatrix}.$$

Once the discrete integral values $R_c(\Delta t), R_c(2\Delta t), \cdots, R_c(p\Delta t)$, and $R_s(\Delta t), R_s(2\Delta t)$, $\cdots$, $R_s(p\Delta t)$ are computed via Eq. (13), substituting them into Eq. (6) yields $q_p(t_{i+1})$, $q_p(t_{i+2}), \cdots, q_p(t_{i+p})$ within the time-step group. Combined with the discrete values of $q_c(t)$ from Eq. (4), this completes the solution to Eq. (1) across all steps in the group.

4. NUMERICAL EXAMPLE

3.1 Structural model and seismic ground motion

A three-story shear-type building is considered to illustrate the proposed time-step group integration method. The structure features rigid floor diaphragms with lumped masses and story stiffnesses listed in Table 1. It is found that the natural vibration periods of the structure are T_1=0.394 s, T_2=0.321 s, and T_3=0.139 s, respectively. Neglecting damping effects, the governing equations of motion under seismic excitation are expressed as:

$$\begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 & \\ -k_2 & k_2+k_3 & -k_3 \\ & -k_3 & k_3 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}=-\begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}\ddot{u}_g(t), \quad (15)$$

where $\ddot{u}_g(t)$ denotes the ground acceleration time history.

Table 1 The mass and stiffness properties of the building

	1	2	3
m (kg)	6.2×10^5	5.4×10^5	1.5×10^4
k (N/m)	5.6×10^8	4.8×10^8	4.0×10^6

The El Centro ground motion (Array #9, 1940 US Imperial Valley earthquake, N-S component) is selected as the input excitation. Its acceleration time history and response spectrum are shown in Figs. 2 and 3, respectively. The recorded motion has a peak ground acceleration (PGA) of 2.75m/s^2 and a predominant period of approximately 0.25s and 0.45s.

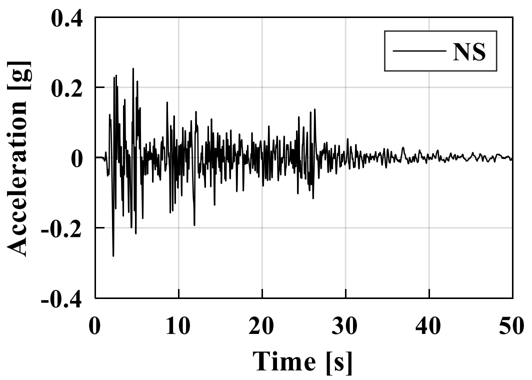

Fig. 2 The El Centro ground acceleration

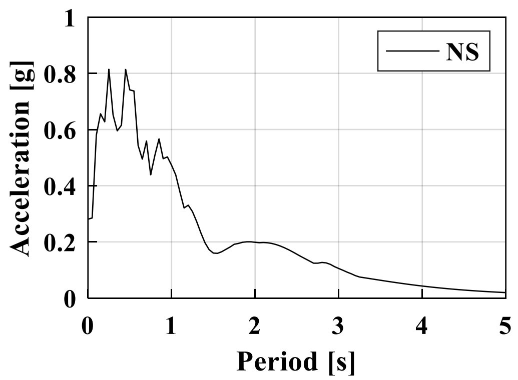

Fig. 3 Acceleration response spectrum

3.2 Seismic response analysis results

The proposed time-step group integration method was applied to analyze the seismic response of the aforementioned structure. All time-step groups were configured with h = 0.1 s and p = 10, resulting in a time step size of $\Delta t = h/p$ = 0.01 s, consistent with the sampling interval of the El Centro ground motion. Notably, the shortest natural period of the structure is T_3 = 0.139 s, yielding a critical ratio of $\Delta t/T_3$ = 0.072 — a stringent test condition that validates the method's stability and precision.

To rigorously verify the method's accuracy, comparative analyses were conducted using the Newmark linear acceleration method (γ = 1/2, β = 1/6) under two configurations:

High-Precision Benchmark: A refined time step of Δ*t* = 0.0001s was adopted to generate a reference solution, regarded as the "exact" result for error quantification.

Equal-Step Comparison: A matching time step of Δ*t* = 0.01s was used to enable direct comparison with the proposed method.

This dual validation strategy isolates the algorithmic error of the time-step group method from temporal discretization errors, providing an objective assessment of its inherent precision. Figs. 4 and 5 present the displacement and acceleration responses of all floors within 0–15 s interval under the El Centro excitation. The results demonstrate that the proposed method achieves near-perfect agreement with the reference solution, exhibiting negligible deviations. While Newmark's method also aligns well with the reference solution, its displacement predictions show superior accuracy compared to acceleration responses. Notably, both methods employed identical time steps (Δ*t* = 0.01s), confirming their reliability in simulating early-stage seismic responses under this temporal resolution.

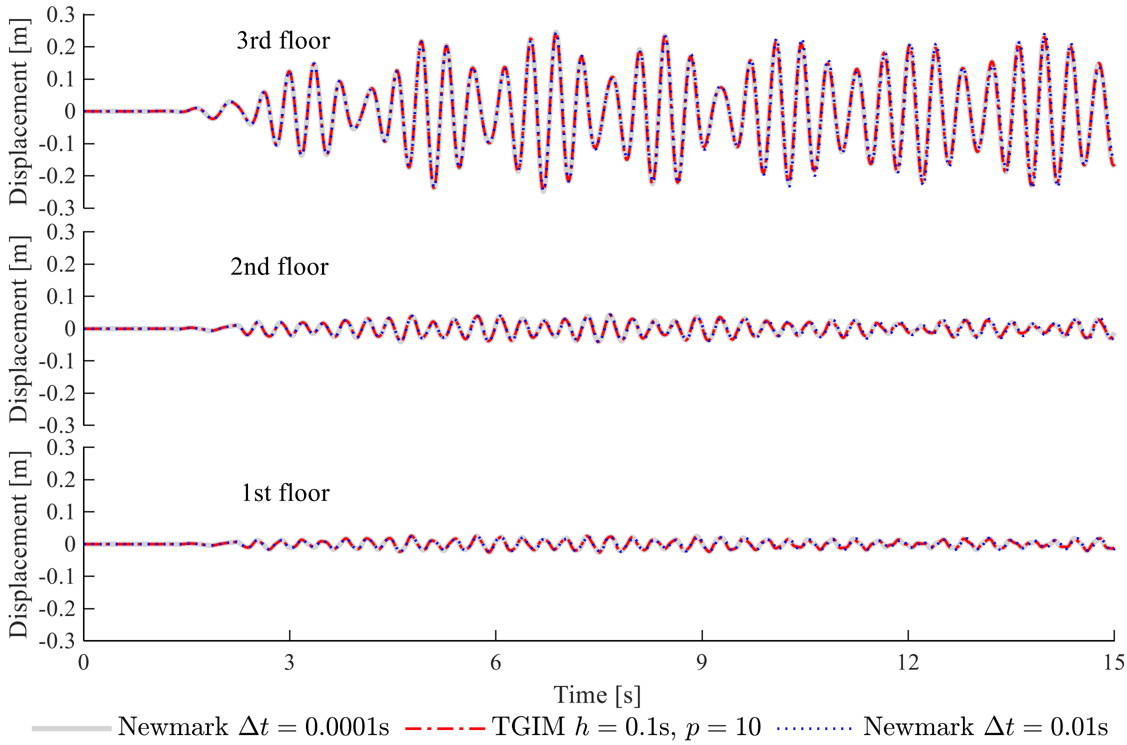

Fig. 4 Comparison of displacement responses of the reference structures (0-15s)

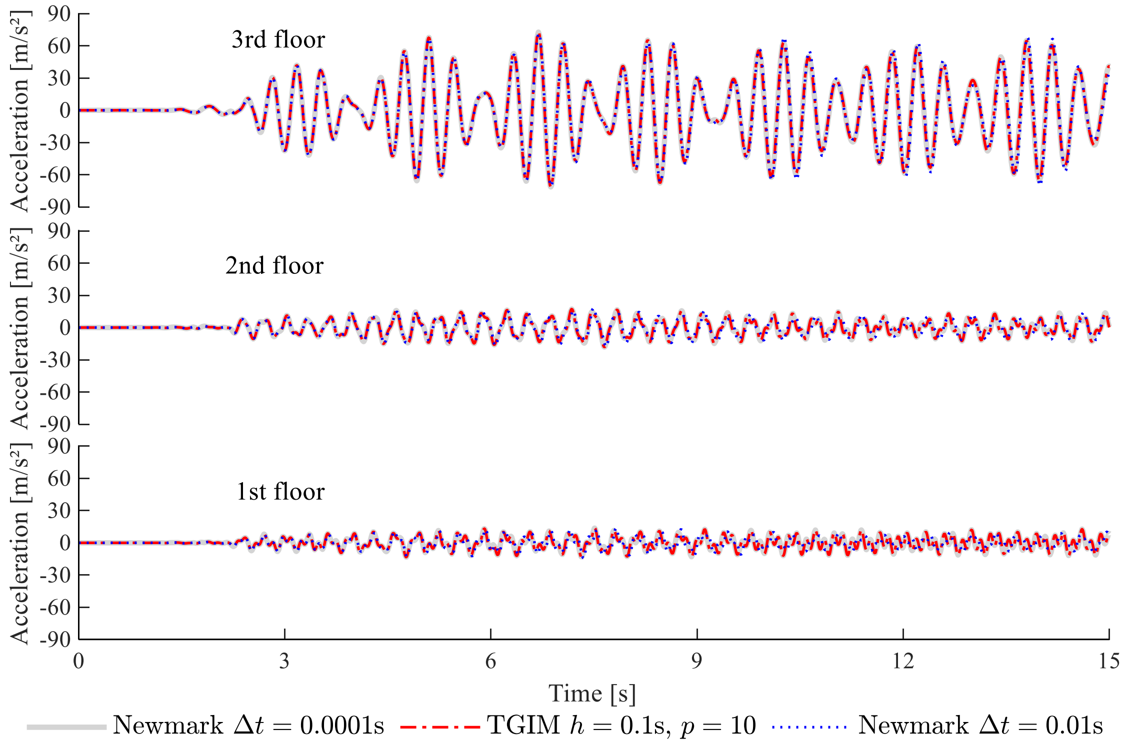

Fig. 5 Comparison of acceleration responses of the reference structures (0-15s)

Figs. 6 and 7 further compare the displacement and velocity responses of the top floor during the 45–50 s interval. The proposed method maintains precise agreement with the reference solution, whereas Newmark's method manifests significant amplitude decay and period elongation. This divergence originates from the numerical dissipation inherent to Newmark's formulation, which accumulates errors over extended durations—a limitation systematically mitigated by the group integration framework.

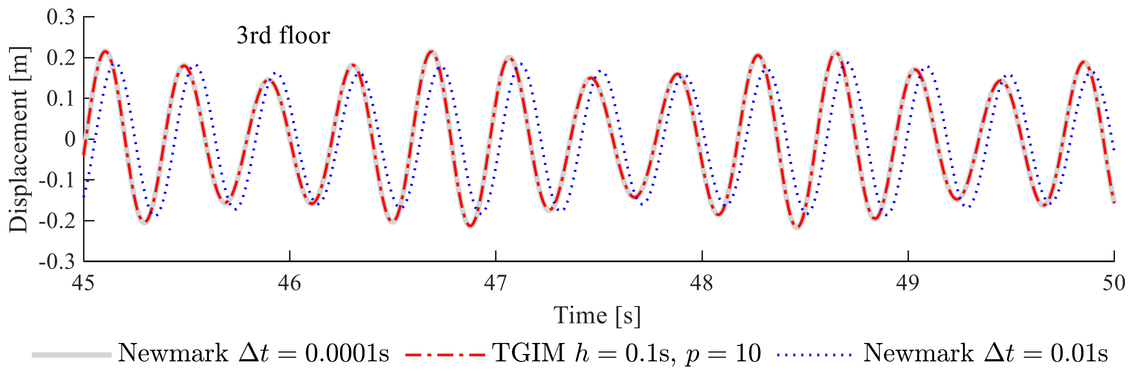

Fig. 6 Displacement response of the top floor over 45-50s interval

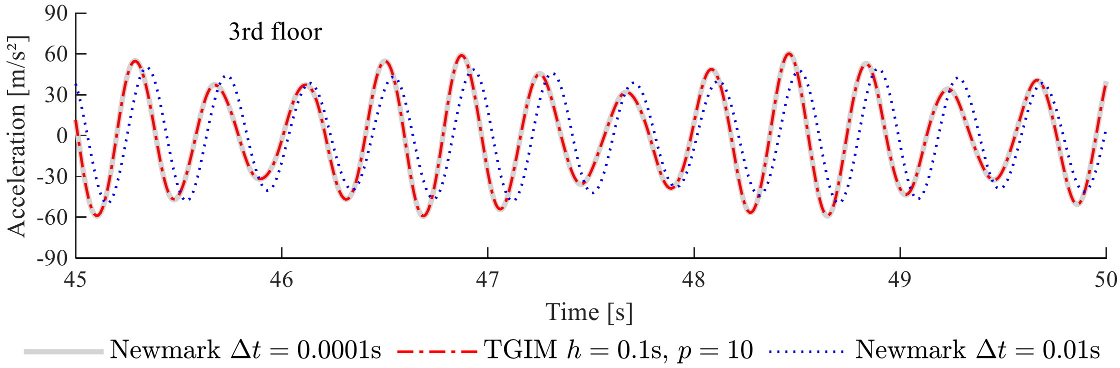

Fig. 7 Acceleration response of the top floor over 45-50s interval

5. CONCLUSIONS

This study presents a time-step group integration method that advances seismic response analysis through two fundamental innovations. First, the method deviates from conventional step-by-step approaches by solving groups of consecutive time steps collectively through matrix operations, thereby reducing error propagation inherent in sequential computations. Second, it eliminates the necessity for direct evaluation of complex integrals through a derivative transformation strategy, overcoming a persistent challenge in existing higher-order time integration methods.

Numerical validation using a three-story shear structure subjected to the El Centro ground motion demonstrates the method's efficacy. When employing a time-step group size of h=0.1s and p=10, which actually determines Δt = 0.01s for seismic response analysis, the method maintains stability and accuracy even with a critical $\Delta t/T_3$ ratio of 0.072, where T_3 = 0.139s represents the structure's shortest natural period. Comparative analyses with the Newmark linear acceleration method reveal that, under equal time steps (Δt = 0.01s), both methods initially yield reliable results. However, during the latter interval of the earthquake duration, the proposed method retains precise agreement with the reference solution, while the Newmark method exhibits noticeable amplitude decay and period elongation—a consequence of accumulated numerical dissipation errors.

These results highlight the method's capability to maintain accuracy over extended durations without sub-step iterations, offering practical advantages for seismic simulations requiring long-term response predictions. The matrix-based group-solving framework also shows inherent potential for parallel computing implementations to further enhance efficiency. Future work will explore the method's adaptability to nonlinear structural systems and stochastic seismic excitations, building upon the validated linear system framework established in this study.

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